The proportional values of α, β, γ in (5) and (6) are given by

$$\frac{\alpha'}{m_1n_2 - m_2n_1} = \frac{\beta'}{n_1l_2 - n_2l_1} = \frac{\gamma'}{l_1m_2 - l_2m_1} \tag{9}$$

and those in (2) and (3) by

$$\frac{\alpha'}{b^3c^3} = \frac{\beta'}{a^3c^3} = \frac{\gamma'}{a^3b^3} \quad (10), \qquad \text{or} \qquad \alpha' \Big/ \frac{1}{a^3} = \beta' \Big/ \frac{1}{b^3} = \gamma' \Big/ \frac{1}{c^3} \quad (11),$$

satisfying (4) and proving the theorem.

The fundamental triangle is in perspective with two other Brocard triangles, the centers of perspective being $1/b^2$, $1/c^2$, $1/a^2$; $1/c^2$, $1/a^2$, $1/b^2$.

Also solved by J. W. Clawson, C. P. Sousley, and S. W. Reaves.

509. Proposed by NORMAN ANNING, Chilliwack, B. C.

A picture whose coördinates are (0, 0), (50, 0), (50, 50), and (0, 50) is repeated on a smaller scale as part of itself with the coördinates (7, 0), (31, 7), (24, 31), (0, 24). Locate the vanishing point.

Solution by S. W. Reaves, University of Oklahoma.

Let P, P_1, P_2, $\cdots$ denote the picture and its successive images, and let O_1, O_2, O_3, $\cdots$ be the successive images of the first vertex. We shall find the vanishing point by determining the limiting position of O_n as n increases indefinitely.

It readily follows from the data of the problem that the side of P_1 is one half that of P and makes with it an angle $\theta = \tan^{-1} 7/24$. It is clear that the side of P_n bears the same relations to the side of P_{n-1} for all values of n.

By projecting the successive segments OO_1, O_1O_2, O_2O_3, $\cdots$ on the coördinate axes, it readily follows that the coördinates of the limiting position of O_n are given by the following infinite series:

$$x = 7 + \frac{7}{2}\cos\theta + \frac{7}{2^2}\cos 2\theta + \cdots + \frac{7}{2^n}\cos n\theta + \cdots,$$

$$y = \frac{7}{2}\sin\theta + \frac{7}{2^2}\sin 2\theta + \cdots + \frac{7}{2^n}\sin n\theta + \cdots.$$

Multiplying the second equation through by $i = \sqrt{-1}$, adding the result to the first equation, denoting $\cos\theta + i\sin\theta$ by v, and remembering that $\cos n\theta + i \sin n\theta$ is by De Moivre's formula equal to v^n, we have

$$x + iy = 7\left[1 + \frac{v}{2} + \left(\frac{v}{2}\right)^2 + \cdots + \left(\frac{v}{2}\right)^n + \cdots\right].$$

Using the formula for the sum of a geometric progression and then replacing v by its value

$$\frac{24}{25} + i\frac{7}{25},$$

we have

$$x + iy = \frac{364}{29} + i\frac{98}{29}.$$

Hence, the coördinates of the required vanishing point are (364/29, 98/29).

Also solved by J. B. Reynolds and W. R. Ransom.

CALCULUS.

424. Proposed by OSCAR S. ADAMS, Washington, D. C.

What is the value of

$$\frac{\Gamma'(1)}{\Gamma(1)} - \frac{\Gamma'(\frac{1}{2})}{\Gamma(\frac{1}{2})}.$$